DIFFERENTIAL GEOMETRY II MIDTERM EXAMINATION

Total marks: 25

Attempt all questions

Time: 2 hours (10 am - 12 noon)

(1) Show that the real projective space (of dimension n) is a smooth manifold (define the set, topology and smooth structure and check the details). (5 marks)

(2) Prove that the orthogonal group $O(n)$ is a manifold. Also show that it is a Lie group. (5+5 marks)

(3) Prove that any smooth vector field on a compact smooth manifold is complete (Hint: use the smooth dependence of the solution of a system of ODE's on the initial value) (5 marks)

(4) Give an example of a smooth vector field on $\mathbb{R}$ which is not complete. (5 marks)